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Data-driven computing for snap-through problems

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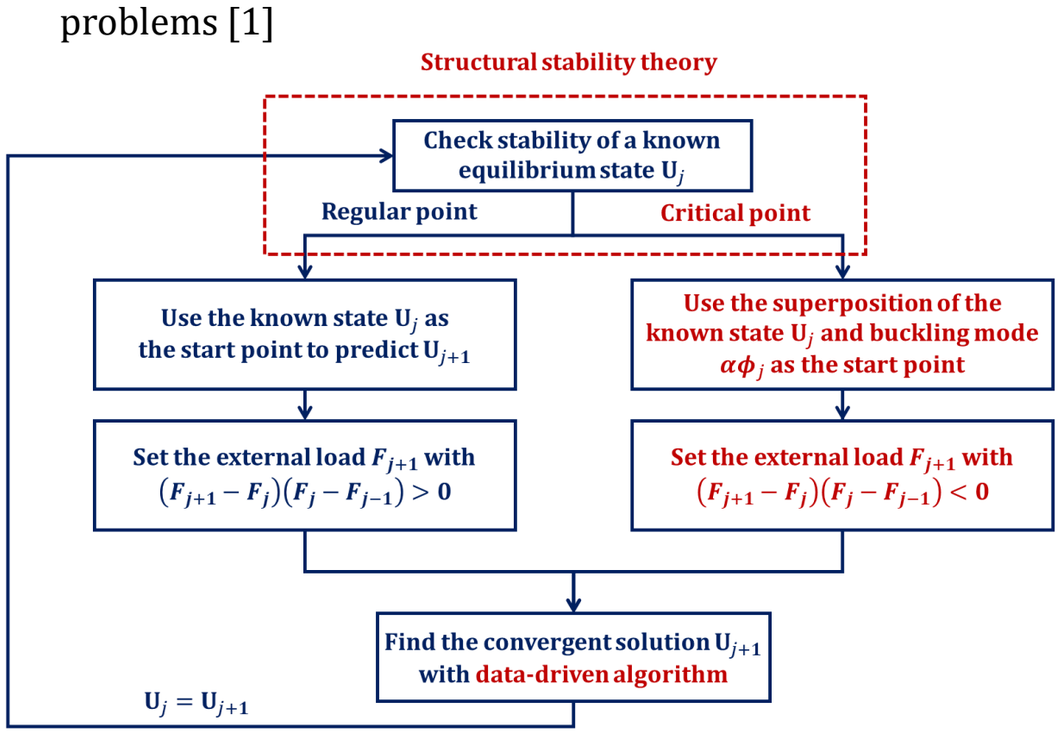
Abstract

The distance-minimizing data-driven computing is an emerging field of computational mechanics [1], which reformulates the classical boundary-value problem as a discrete continuous optimization problem. However, the optimization problem becomes non-convex due to the limit equilibrium instability in snap-through phenomenon, which makes it challenging to obtain a desired convergent solution, especially near critical points. Towards this end, we propose a data-driven computational framework by virtue of the structural stability theory for analyzing snap-through problems [2]. First, an auxiliary linearized perturbed system is established at each equilibrium state (i.e., convergent data-driven solution), which can be solved with the stiffness-based method and data-based method. Then, a stability indicator is employed to detect the critical points and the corresponding buckling mode [3], which is utilized to construct a proper start point to trace the equilibrium path in the neighborhood of critical points. The reliability of the proposed framework is proved even in snap-through problem with nonlinear constitutive behavior.

Methodology and Context

- Data-driven computational framework for snap-through problems [1]

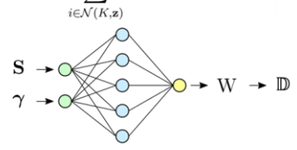


- Two methods for solving a linearized perturbed system in structural stability theory

Stiffness-based method

- **Approximate the tangent constitutive tensor with data-fitting or data-regression**

$$\mathcal{T}(\mathbb{D},\mathbb{D}_0)=\sum_{i\in\mathcal{N}(K,\boldsymbol{x})}(\mathbf{S}_i'-\mathbb{D}:\boldsymbol{\gamma}_i'-\mathbb{D}_0)^2$$



- **Obtain $\Delta\hat{\mathbf{q}}$ with tangent stiffness matrix**

$$\sum_{b=1}^{N^t}\mathbf{K}_{m(ab)}\cdot\Delta\hat{\mathbf{q}}_b=\mathbf{F}_{p(a)},\quad a=1,\dots,N^t$$

Data-based method

- **Construct a distance-minimizing function**

$$\Pi_p=\mathscr{F}(\boldsymbol{\gamma}+\Delta\hat{\boldsymbol{\gamma}},\mathbf{S}+\Delta\hat{\mathbf{S}})-\sum_{a=1}^{N^t}\Big(\int_\Omega\mathbf{S}:\mathbf{B}_{nl}(\Delta\hat{\mathbf{u}},N_a)+\Delta\hat{\mathbf{S}}:(\mathbf{B}_l(N_a)+\mathbf{B}_{nl}(\mathbf{u},N_a))\,\mathrm{d}\Omega-\int_{\partial\Omega}N_a\mathbf{f}_p\,\mathrm{d}\Gamma\Big)\cdot\boldsymbol{\eta}_a$$

- **Obtain $\Delta\hat{\mathbf{q}}$ with an auxiliary data-driven algorithm**

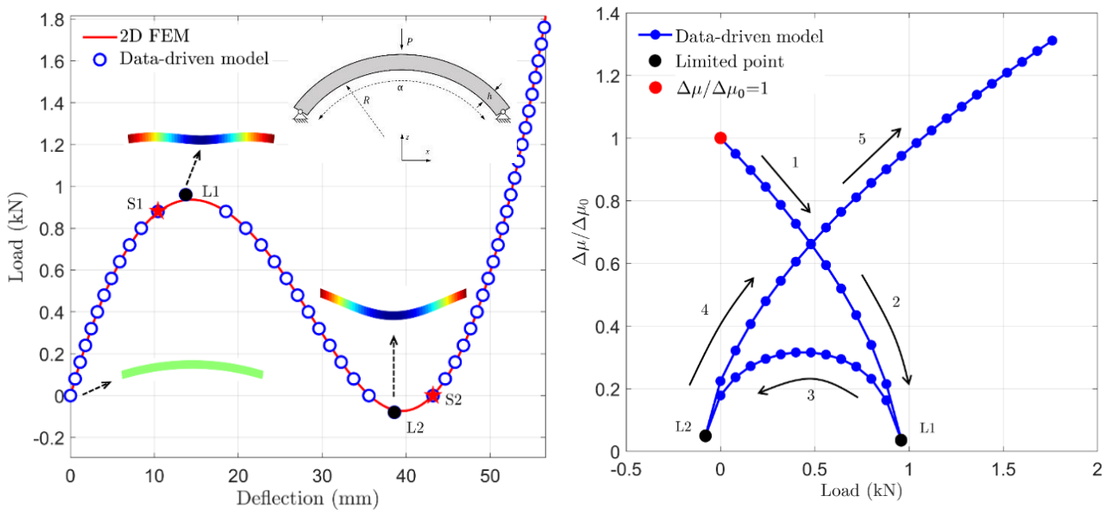
$$\delta\Delta\hat{\mathbf{u}}:\ \int_\Omega(\boldsymbol{\gamma}+\boldsymbol{\gamma}_l(\Delta\hat{\mathbf{u}})+\boldsymbol{\gamma}_{nl}(\mathbf{u},\Delta\hat{\mathbf{u}})-\hat{\boldsymbol{\gamma}}^*):\mathbb{C}:(\boldsymbol{\gamma}_l(\delta\Delta\hat{\mathbf{u}})+\boldsymbol{\gamma}_{nl}(\mathbf{u},\delta\Delta\hat{\mathbf{u}}))-\mathbf{S}:\boldsymbol{\gamma}_{nl}(\boldsymbol{\eta},\delta\Delta\hat{\mathbf{u}})\,\mathrm{d}\Omega=0$$

$$\delta\Delta\hat{\mathbf{S}}:\ \int_\Omega\delta\Delta\hat{\mathbf{S}}:\mathbb{C}^{-1}:(\mathbf{S}+\Delta\hat{\mathbf{S}}-\hat{\mathbf{S}}^*)\mathrm{d}\Omega=\int_\Omega\delta\Delta\hat{\mathbf{S}}:(\boldsymbol{\gamma}_l(\boldsymbol{\eta})+\boldsymbol{\gamma}_{nl}(\mathbf{u},\boldsymbol{\eta}))\,\mathrm{d}\Omega$$

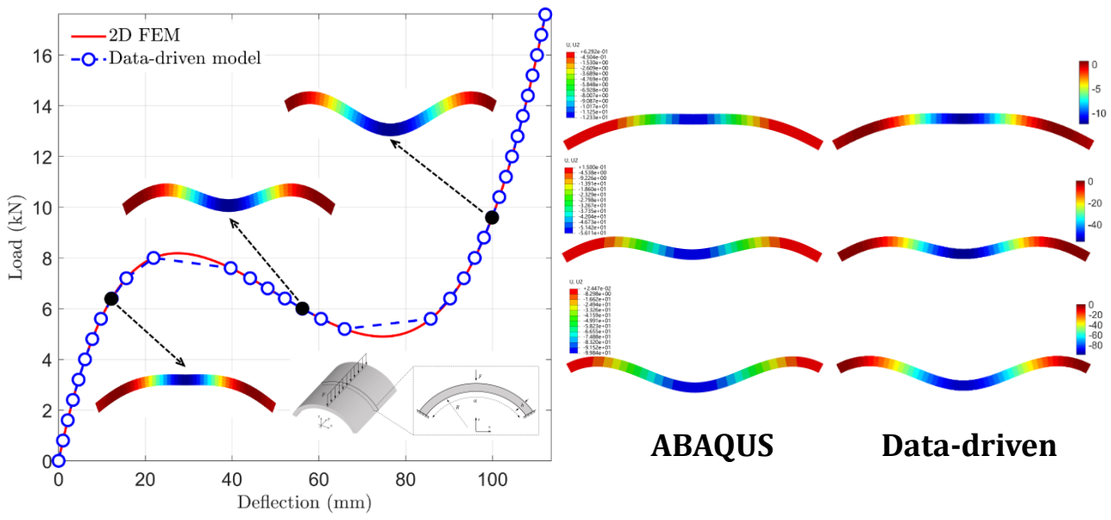
$$\delta\boldsymbol{\eta}:\ \int_\Omega\mathbf{S}:\boldsymbol{\gamma}_{nl}(\Delta\hat{\mathbf{u}},\delta\boldsymbol{\eta})+\Delta\hat{\mathbf{S}}:(\boldsymbol{\gamma}_l(\delta\boldsymbol{\eta})+\boldsymbol{\gamma}_{nl}(\mathbf{u},\delta\boldsymbol{\eta}))\,\mathrm{d}\Omega-\int_{\partial\Omega}\delta\boldsymbol{\eta}\cdot\mathbf{f}_p\,\mathrm{d}\Gamma=0$$

Results

- The load-displacement curve (left) and stability indicators (right) of an elastic curved beam



- The load-displacement curve (left) and deformation distribution (right) of the hyper-elastic cylindrical shell.



The numerical results show good accuracy and reliability of the proposed data-driven computational framework

Conclusion and perspectives

In this work, a data-driven computational framework for the snap-through problem is proposed, consisting of the structural stability theory and the distance-minimizing data-driven algorithm. It shows ability to correctly predict the snap-through behavior even with material nonlinearity and strong geometric nonlinearity. The proposed framework has potential application in snap-through problems with composite materials, e.g., hydrogel or dielectric material, which pose multifunction and design flexibility.

References

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[3] H. J. Weinitschke, On the calculation of limit and bifurcation points in stability problems of elastic shells, ***Int. J. Solids Struct.*** 21 (1985) 79-95.