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FACULTE DES SCIENCES BEN M'SIK
UNIVERSITÉ HASSAN II DE CASABLANCA

Laboratoire de Mécanique et d'Acoustique

LaMé Laboratoire de Mécanique Gabriel Lamé

WUHAN UNIVERSITY School of Civil Engineering

Ecole Nationale d'Ingénieurs de Tunis

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## Hermite-type PIM Method for analysis of rectangular functionally graded material thin plates buckling


Amina Hammou[a], Youssef Hilali[a], Said Mesmoudi[c], Radouane Boujmal[a], Oussama Bourihane[a]

[a] Laboratory of Mechanical Engineering, Faculty of Sciences and Technology Fez, Sidi Mohamed Ben Abdellah University, Fez, Morocco
[b] Hassan First University of Settat, Ecole Nationale des Sciences Appliquées, LISA Laboratory, Berrechid 26100, Morocco



## Abstract

This work intends to elaborate a Meshfree approach of a thin plate made by a mixture of a ceramic metal Functionally Graded Material (FGM), which utilises meshless methods PIM and HPIM for the nonlinear buckling analysis. Where, the Classical Plate Theory (CPT) is utilised to introduce the plate kinematics. The nonlinear static equilibrium differential equations and related boundary conditions of the plate are laid through the principle of Stationary Potential Energy. The in-plane displacements, the transverse displacement and its derivatives are estimated successively by PIM method and HPIM, in order to get the discrete weak form. By transforming the equations obtained to an eigenvalue problem, one can get the critical loads resulting from buckling and the associated mode shapes. Examples of rectangular FGM plates exposed to uniaxial and biaxial in-plan loadings are explored to show the efficiency of the proposed approach. Results of the presented approach, FEM and some published data in the literature are compared together.


## Solution Methodology

Let us consider a thin ceramic-metal FGM rectangular plate of in-plane dimensions a and b and thickness h, having rectangular cross-section with Cartesian coordinate system (O, x, y, z) established in the middle plane and having the origin O at one corner.

According to the Classical thin Plate Theory [4], the 3D displacement components of a point M(x, y, z) are expressed as functions of the mid-plane displacements u, v, w and the partial derivatives of the transverse deflection with respect to the spatial coordinates x and y, as:

$$u_M(x,y,z) = u(x,y) - zw_{,x}(x,y)$$
$$v_M(x,y,z) = v(x,y) - zw_{,y}(x,y)$$
$$w_M(x,y,z) = w(x,y)$$

With considering Dirichlet boundary conditions $u = u_d$ imposed on the boundary $\Gamma_u$, the MPE principle leads to the system :

$$\begin{cases} \int_A \delta \boldsymbol{E}^T \boldsymbol{N} + \delta \boldsymbol{\kappa}^T \boldsymbol{M} dA - \lambda \int_{\Gamma_t} \delta \boldsymbol{u}^T \bar{\boldsymbol{f}} d\Gamma_t = 0 \\ \\ \boldsymbol{N} = \bar{\boldsymbol{D}}(C_0 \boldsymbol{E} + C_1 \boldsymbol{\kappa}) \\ \boldsymbol{M} = \bar{\boldsymbol{D}}(C_1 \boldsymbol{E} + C_2 \boldsymbol{\kappa}) \\ \\ \boldsymbol{E} = (\boldsymbol{H_1} + \frac{1}{2}\boldsymbol{A(\theta)})\boldsymbol{\theta} \\ \boldsymbol{\kappa} = \boldsymbol{H_2 \theta} \end{cases}$$

where N and M represent the resultants in-plane forces and bending moments respectively. E is the generalized membrane strain, κ is the bending strain, and $\bar{f}$ is the external force vector.

Then, the discrete form of stability equations is obtained using the PIM and HPIM functions :

$$\boldsymbol{u(x)} = \sum_{I=0}^{n} \boldsymbol{\phi_I(x) u_I}$$

$$\boldsymbol{v(x)} = \sum_{I=0}^{n} \boldsymbol{\phi_I(x) v_I}$$

$$\boldsymbol{w(x)} = \sum_{I=0}^{n} \boldsymbol{\psi_I(x) w_I} + \boldsymbol{\psi_{Ix}(x) w_{,xI}} + \boldsymbol{\psi_{Iy}(x) w_{,yI}}$$

$$\boldsymbol{w,x(x)} = \sum_{I=0}^{n} \boldsymbol{\psi_{I,x}(x) w_I} + \boldsymbol{\psi_{I,xx}(x) w_{,xI}} + \boldsymbol{\psi_{I,xy}(x) w_{,yI}}$$

$$\boldsymbol{w,y(x)} = \sum_{I=0}^{n} \boldsymbol{\psi_{I,y}(x) w_I} + \boldsymbol{\psi_{I,yx}(x) w_{,xI}} + \boldsymbol{\psi_{I,yy},(x) w_{,yI}}$$

Solving the resulting eigenvalue problem makes it possible to establish the critical buckling load:

$$(\boldsymbol{K} + \lambda \boldsymbol{K}g)\,\boldsymbol{u} = \boldsymbol{0}$$

where ***K*** and ***Kg*** represent the elastic and geometric stiffness matrices.

## Context

FGM is microscopically inhomogeneous composite material in which the material properties are smoothly and continuously vary following a specific direction(s) [1, 2]. This is obtained by varying the volume fraction of the constituents. Buckling of FGM plates have been considered by the scientists and engineers as a new area for researches [1].

In the framework of this contribution, the meshfree method is adopted, in one hand, to overcome the FEM method difficulties when we investigate a specific topic concerning structures made of non-homogeneous materials. On the other hand, to generalize the use of the appraoch proposed by Bourihane et al. [3] in their recent work. Therefore, in this paper, we apply PIM and HPIM meshless computational method to study the nonlinear buckling behavior of thin plates with simple geometry having regular boundaries. To study this phenomena, it is necessary to use mathematics to comprehensively understand and quantify it. Most of physical phenomenon are described using partial differential equations. However, for a computer to solve these equations, numerical techniques have been developed over the last few decades and one of the most prominent today is the finite element method but it has mesh-related difficulties such as mesh distortion and remeshing. That's why the developing of meshless methods is necessary. That's why we aim to use a hermite-type PIM for buckling analysis of FGM thin plates.

## Numerical results and discussion

Several test cases of rectangular plates with uniform thickness exposed to different loadings and boundary conditions are investigated at this step (figure 1). For the PIM approximation we use a bilinear basis function and a cubic polynomial basis ones for HPIM, in order to evaluate the variables field.

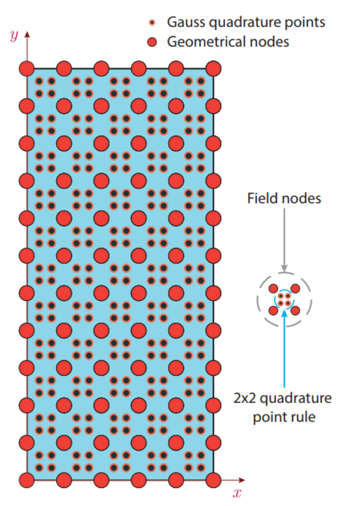
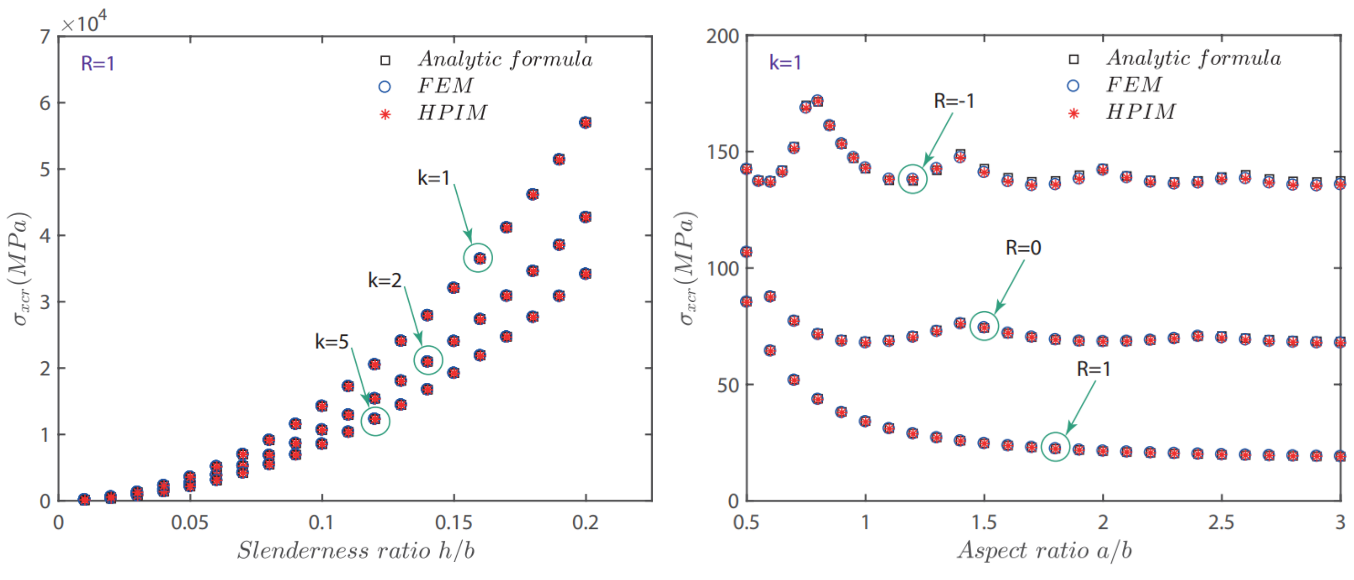



Figure 1 : Structured discretization that cover the problem domain

Example : Biaxial buckling

We consider a rectangular straight FGM plate under in-plane uniaxial compression $\sigma_x$ which is assumed proportional to a load parameter λ while fixing the power-law index to *k* = 1. Looking for the evaluation of the loading conditions impact on the effectiveness of the proposed approach in the computation of the critical buckling load, we plot in figures 2(a) and 2(b) the variation of the critical buckling load according to *a/b* and *h/b* namely successively the aspect and the slenderness ratio for the uniaxial compression (*R* = 0), compression-compression (*R* = 1) and compression-traction loading (*R* = -1).




(a) Versus the slenderness ratio h/b.

(b) Versus the aspect ratio a/b.

Figure 2: Critical buckling loads of an FGM thin plate according to the loading conditions.

This section present numerical experiments that are utilised to show the influence of various parameters. The obtained results for the three examples by meshless HPIM method are compared always with those of the considered reference available data and FEM ones, namely: analytical formula [5] and/or FEM [3]. These results show a very good agreement compared to the references data.

## Conclusion and perspectives

In this work, we applied the PIM and HPIM to study the nonlinear buckling response of FGM plates. The Classical Plate Theory (CPT) is used to describe the plate kinematics. Numerical experiments have shown that the utilised approach gives results that are with one voice with those obtained by the analytical solutions and/or the FEM. Hence, we deduce that the proposed approach can successfully estimate the nonlinear buckling behavior of FGM rectangular plates . As a perspectives, we intend to develop meshless methods which exceed these limits and will be valid for other structures;

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