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High order RPIM algorithm for transition analysis of thermally induced flow in an annulus of two concentric circular cylinders

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Abstract

We present a natural convection mesh-free solution regarding closed two-dimensional ring of two concentric circular cylinders with different air-filled operating conditions. The numerical technique uses the Radial Point Interpolation Method (RPIM) shape functions to discretize the weak form of the governing equations and a high order continuation procedure to compute the nonlinear solution.
The thermophysical properties of the working fluid are assumed to be constant except the density variations causing a body force term in the momentum equation. No-slip boundary conditions are applied at all enclosure boundaries.
The results are presented in terms of isotherms and average equivalent heat conductivity number obtained, as well as a more quantitative comparison between the results obtained from the current computer code and those available in the literature.

Methodology

To achieve the objectives of this work, we will use an approach based on the Meshfree method. Most of the existing numerical works concerning thermal convection are based on the Boussinesq approximation for: connect temperature and density changes and for couple the temperature field and flow one.
Therefore, the dimensionless form of the governing equations for a stable natural convection flow using conservation of mass, momentum and energy can be written as follows (Eq.1) :

$$\begin{aligned}
\frac{\partial u}{\partial x} &+ \frac{\partial u}{\partial y} &=& \; 0\\
u\frac{\partial u}{\partial x} &+ u\frac{\partial u}{\partial y} &=& -\frac{\partial P}{\partial x} + P_r\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\\
u\frac{\partial u}{\partial x} &+ u\frac{\partial u}{\partial y} &=& -\frac{\partial P}{\partial y} + P_r\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + R_a P_r \Theta\\
u\frac{\partial \Theta}{\partial x} &+ u\frac{\partial \Theta}{\partial y} &=& \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right)
\end{aligned}$$

Ra : Rayleigh number; Pr: Prandtl number and U ,V: Dimensionless velocity components in the x and y directions

- ❖ We use the continuity equation (Eq.1a)as the constraint due to mass conservation.
- ❖ In order to solve (Eqs.1b–1d), a penalty technique, which is a standard method for solving incompressible viscous flows, is used.
- ❖ The pressure *P* can be eliminated from the equilibrium and energy equations (Eqs. 1b–1d) by the constraint equation derived using a penalty parameter *γ* and the incompressibility criteria.

$$P = -\gamma\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Uses the Radial Point Interpolation Method (RPIM) shape functions to discretize the weak form of the governing equations and a high order continuation procedure (HOCM) to compute the nonlinear solution.
For the boundary conditions: $U = V = 0$ on all walls ;$\Theta = 0$ on colded wall ;$\Theta = 1$ on heated wall.

Context

The absence of studies concerning natural convection on different types of circular solids filled with fluids motivated us to undertake this investigation.

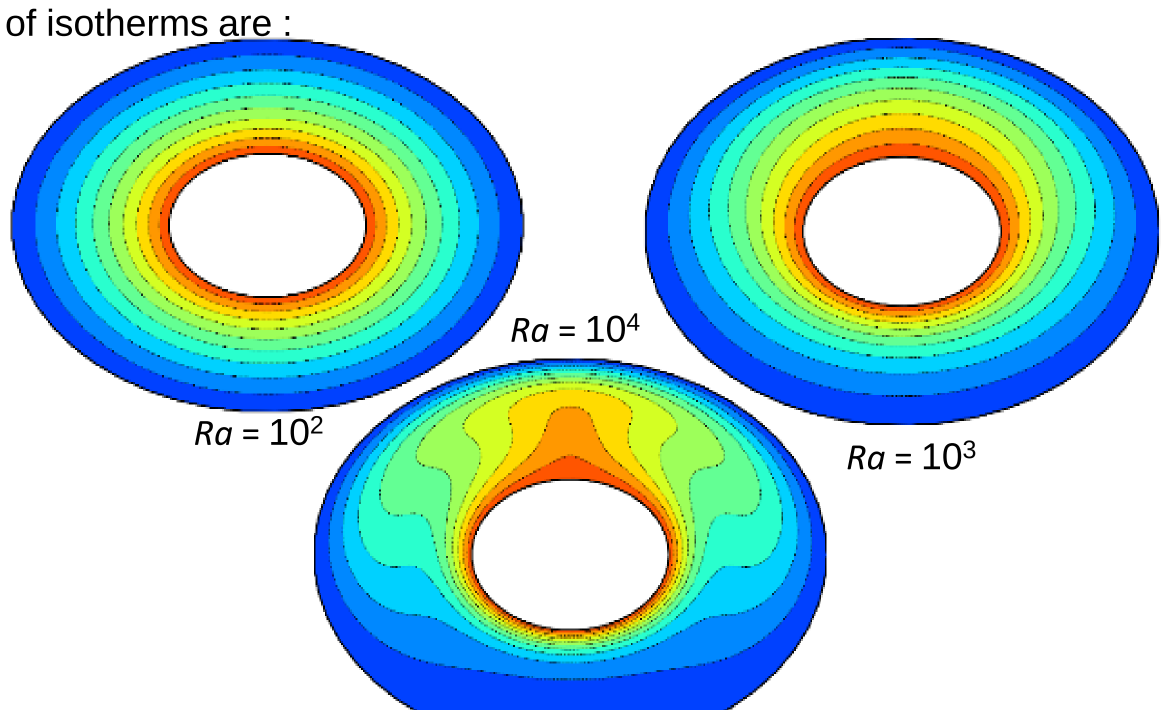
- ✓ Let's consider thermal convection in a (2D) ring made of two concentrically circling cylinders.
- ✓ Di is the inner cylinder's diameter, while L is the distance between both concentric cylinders.
- ✓ The temperatures of the inner and outer cylinders are Th and Tc respectively, with the condition (Th > Tc).
- ✓ The ring is filled with air, so that the Prandtl number (Pr) is about 0.71.
- ✓ All thermo-physical properties of the Newtonian fluid are assumed to be constant, except for density variations causing a body force term in the momentum equation.
- ✓ It is assumed that the fluid is viscous and compressible in thermal convection and that it is in thermal equilibrium and that there is no slip between the seals and the fluid.

Results

The performance and accuracy of the proposed RPIM-HOCM Solver is investigated by comparing the computed values of the average equivalent conductivity to those obtained in the framework of previous research works done by others researchers using other different numerical computational methods: $\tilde{k}_{eqo} = \frac{-rr ln(rr)}{\pi(rr-1)}\int_0^{\pi}\frac{\partial\Theta}{\partial r}d\theta$ $\tilde{k}_{eqi} = \frac{-ln(rr)}{\pi(rr-1)}\int_0^{\pi}\frac{\partial\Theta}{\partial r}d\theta$

Ra	Reference	Numerical method	Inner cylinder $\tilde{K}$eqi	Outer cylinder $\tilde{K}$eqo
10^2	Present study Ho –Minh et al [1] Shu [2]	RPIM G-IRBFN FDQ	0.984 1.001 1.001	1.007 1.001 1.001
10^3	Present study Ho –Minh et al [1] Shu [2]	RPIM G-IRBFN FDQ	1.063 1.080 1.082	1.091 1.079 1.082
10^4	Present study Ho –Minh et al [1] Shu [2]	RPIM G-IRBFN FDQ	1.941 1.967 1.979	1.993 1.953 1.979

The numerical results for different number of Rayleigh in the form of isotherms are :



Conclusion and perspectives

Based on the calculated values of the average equivalent conductivity to those obtained in the framework of previous research work carried out by other researchers using other different numerical calculation methods and the series of simulation which were performed in the range of Rayleigh number $10^2 \leq Ra \leq 10^4$,we can conclude the numerical method was validated and a good agreement was obtained, and for our perspective we will :

- ➢ Explore the stable and transient case of natural convection flows;
- ➢ Extend this work for some complicated cases
- ➢ Use an approach integrating the technique without mesh other in the context of a strong formulation.

References

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