Workshop « Soft Material Models »

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A decoupled homogenization methodology for anisotropic hyperelastic media

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Abstract

This study focusing on performing nonlinear two-scale analysis on composite materials possessing a periodic microstructure, The proposed strategy is based on the assumption that a functional form of the macroscopic constitutive equation is available. To reduce the computational costs to solve a two-scale boundary value problem without losing the distinctive characteristics of the coupled methods, a micro-macro decoupling scheme proposed by Terada et AL [1,2] is employed. This involves conducting a series of numerical material tests (NMTs) on the Representative Volume Element to perform data acquisition, A tensor-based method of parameter identification with the "measured" data in the NMTs is used for identifying the material parameters in the assumed anisotropic hyperelastic constitutive model equation. Once the macro-scale material behavior is successfully fitted with the identified parameters, macro-scale analysis can be performed.

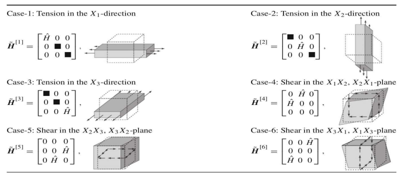
Methodology

The homogenization technique relies on the research introduced by Terada et al. [1] and involves a step-by-step approach that includes:

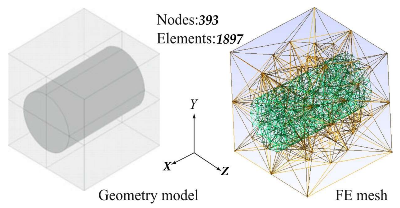
- Choosing an anisotropic hyperelastic macroscopic constitutive law (Homogenized Potential).

$$W = W_{vol}(J,D) + W_{iso}(\overline{I_1},\overline{I_2},a,b) + W_{aniso}(I_4,I_5,I_6,I_7,I_8,c,d,e,f,g,A,B)$$

- Performing numerical simulations on a representative heterogeneous volume element (RVE) consisting of a composite materials(matrix / fibers):
  - ➢ Choosing a random subset of macroscopic displacement gradient **H** (Type and Intensity).



  - ➢ Computing the second Piola-Kirchhoff stress tensor (S_{heter}) by solving the corresponding boundary value problem through integration over the heterogeneous RVE with periodic boundary conditions(PBC).



- Macroscopic coefficient identification for the chosen homogenized potential :
  - ➢ Express the second Piola-Kirchhoff stress tensor as a function of the chosen homogenized potential and material coefficients $\mathbf{p}^{[k]}$:

$$\mathbf{S}_{homog} = \sum_{\alpha=0}^{n_{test}} p^{[k]}\,\mathbf{g}^{[k]}$$; $\mathbf{g}^{[k]}$ are the derivatives of the different terms in the potential,

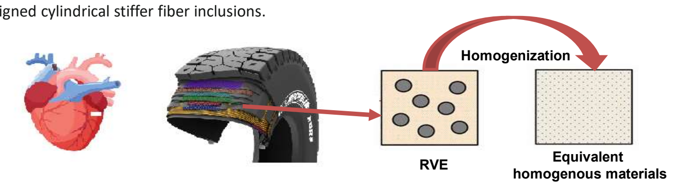
  - ➢ Identify the macroscopic coefficients using a least squares optimization method:

$$\theta(p) = \frac{1}{2}\sum_{\alpha=1}^{n_{test}} \frac{\| S_{homog}^{(\alpha)}(p) - S_{heter}^{(\alpha)} \|^2}{\| S_{heter}^{(\alpha)} \|^2}$$

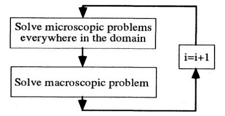
- Calculate the local error of the RVEs between S_{homo} and S_{heter} to evaluate the capability of the theoretical potential to closely approximate the RVE behaviour .

Context

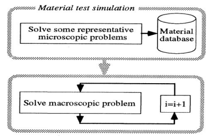
- ➢ Fiber-reinforced rubber-like composites can be physically represented as a pliable matrix material with aligned cylindrical stiffer fiber inclusions.



- ➢ The macro model and the micro-macro approaches (Coupled Method) present challenges due to their high cost and complex calculations and the Lack of link between model parameters and physical microstructure .
- ➢ The decoupled methodology synergistically combines micro-mechanical and macro-mechanical phenomenological approaches, mitigating their respective limitations while leveraging their benefits.



(a) Micro-macro iterative coupling method



(b) Micro-macro iterative decoupling method

Results

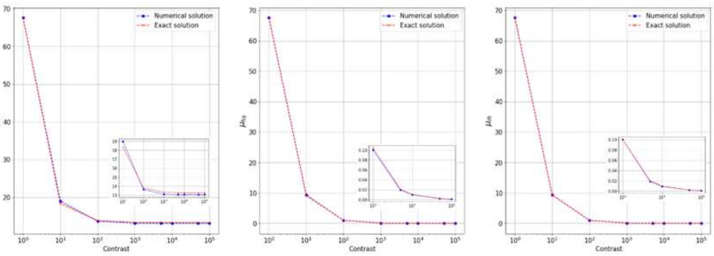
❑ Validation of the homogenization method in the NeoHookean case

Fiber/Matrix constitutive model: NeoHookean (Slightly compressible case)

$$\tilde{W}_{NH,i} = C_{01,i}(I_1 - 3) + D_{1,i}(J-1)^2 \quad \text{With} \quad C_{01,i} = \frac{E_i}{4(1+\nu_i)},\quad D_{1,i} = \frac{E_i}{6(1-2\nu_i)},$$

Theoretical Homogenized Potential: (Slightly compressible case)

$$W = \frac{\mu_{ih}}{2}(\overline{I_1} - 3) + \frac{\mu_n - \mu_{ih}}{2}\left(\frac{2}{\sqrt{\overline{I_4}}} - 3\right) + \frac{\mu_n - \mu_{HS}}{2}\overline{I_4} - \frac{\mu_{ih} - \mu_{HS}}{2}\frac{\overline{I_5}}{\overline{I_4}} + D(J-1)^2$$



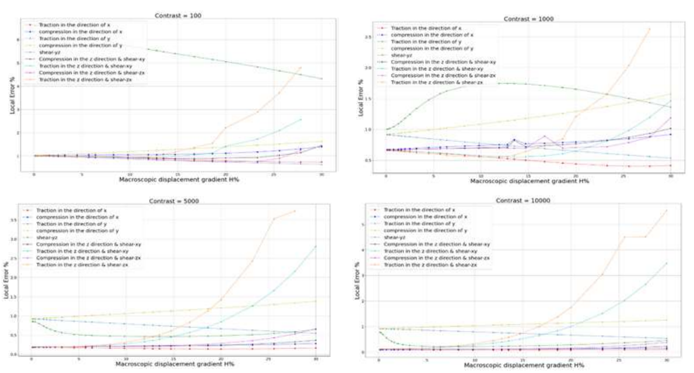
Error between the numerical and analytical coefficients μ_n and μ_{hs} and μ_{ih} for a Poisson coefficient v = 0.49 and for a contrast C ∈[1, 10⁵]

❑ Local Error : slightly compressible hyperelastic model

Fiber/Matrix constitutive model:
- **Fiber: Saint Venant Kirchhoff**
- **Matrix: Mooney Rivlin**

Theoretical Homogenized Potential: (Slightly compressible case)

$$W = D(J-1)^2 + \sum_{i=1}^{3} a_i(I_1-3)^i + \sum_{j=1}^{3} b_j(I_2-3)^j + \sum_{k=2}^{6} c_k(\overline{I_4}-1)^k + \sum_{l=2}^{6} d_l(\overline{I_5}-1)^l + \sum_{m=2}^{6} e_m(\overline{I_6}-1)^m + \sum_{n=2}^{6} f_n(\overline{I_7}-1)^n + \sum_{o=2}^{6} g_o(\overline{I_8}-\xi)^o$$



Local error of Kaliske's transverse isotropic law with respect to the deformation % and Four different contrasts

Conclusion and perspectives

- ➢ The Neo-Hookean hyperelastic potential where the homogenized law is already established In the case of incompressible materials was utilized to confirm the effectiveness of the proposed technique.
- ➢ The proposed method Manifest a strong performance in terms of local error by employing An isotropic hyperelasticity model for the constitutive model of the microstructures, while an orthotropic one is assumed to represent the macroscopic material behavior.
- ➢ Propose a numerical homogenization strategy aiming for the best possible compromise in terms of computational cost and reliability while considering the distinct tension-compression behaviors of fibers .

References

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