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A high-order algorithm for finite transformation elasto-plasticity problems

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Abstract

This study presents an algorithm based on Asymptotic Numerical Method (NAM) techniques and improved vector Padé approximants for the simulation of elastoplastic structures in large deformations. More specifically, the algorithm uses a new class of vectorial Padé approximants instead of a Taylor series representation, in order to accelerate the convergence of the algorithm in the case of finite-transformation elasto-plasticity problems. The efficiency and robustness of this algorithm are tested on examples of 2D structures, resulting in a reduction in the number of tangent matrices required to find the solution, compared with the use of Taylor series.

Keywords:
Asymptotic Numerical Method (ANM), Vectorial Padé approximants, Finite Element Method.

Methodology

The equilibrium in the Lagrange configuration is written in the matrix form:

$$\int_{\Omega_0} \langle \delta L \rangle^t [A(f)]\{\tau\}\, d\Omega_0 = C(t) \int_{\partial\Omega_0} \langle \delta v \rangle \{F_{ext}\}\, dS_0$$

The elastic behavior law part, can be derived from the classical relation between the stress vector and the elastic strain rate vector in the following matrix form:

$$\{\tau^J\} = [C^{elas}](\{D\} - \{D^p\})$$

The plastic behavior part is given by the normality condition:

$$\{D^p\} = \dot{\lambda}\left\{\frac{\partial f_y}{\partial \tau}\right\} = \dot{\lambda}\{n\}$$

The plastic multiplier is expressed as :

$$\dot{\lambda} = G(f_y) H(D)$$

The function G regularizes the elastic–plastic transition and the function H regularizes the elastic discharge:

$$G(f_y) = \frac{\eta_3}{\frac{f_y^2}{2\mu}\tau_e + \eta_3\left(\frac{3}{2} + \frac{h}{2\mu}(1+f_y)\right)} \; ; \; H(H-\xi) = \eta_2^2 \dot{t}_c^2 \; ; \; \xi = n : D \; ; \hat{F} = f_y^2 \; ; Den = \frac{\hat{F}}{2\mu}\tau_e + \eta_3\left(\frac{3}{2} + \frac{h}{2\mu}(1+f_y)\right) \; ; \; GDen = \eta_3$$

The new Padé approximants used in this work are computed directly from the vectorial series, in the following forms :

$$\boldsymbol{U}[0,N](a) = \boldsymbol{U}^j + \sum_{m=1}^{N} \frac{\Delta_{N-m}(a)}{\Delta_N(a)} a^m \boldsymbol{U}_m$$

$$\boldsymbol{U}[1,N-1](a) = \boldsymbol{U}^j + a\boldsymbol{U}_1 + \sum_{m=1}^{N-1} \frac{\Delta_{N-1-m}(a)}{\Delta_{N-1}(a)} a^{m+1} \boldsymbol{U}_{m+1}$$

$$\boldsymbol{U}[2,N-2](a) = \boldsymbol{U}^j + a\boldsymbol{U}_1 + a^2\boldsymbol{U}_2 + \sum_{m=1}^{N-2} \frac{\Delta_{N-2-m}(a)}{\Delta_{N-2}(a)} a^{m+2} \boldsymbol{U}_{m+2}$$

Context

The Asymptotic Numerical Method (ANM) [1] has emerged as a computational methodology for tackling nonlinear problems through the utilization of Taylor series expansions [2,3,4]. In contrast to conventional incremental methods, ANM capitalizes on the comprehensive information embedded within a path $a \to U(a)$, thereby surpassing the limitations of point-to-point computations. Notably, ANM offers the flexibility to determine step lengths from the Taylor series, while concurrently enabling the incorporation of convergence acceleration techniques such as the Padé approximant method to widen its validity range.

Results

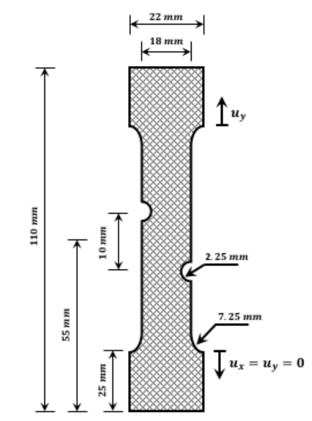

Fig.1: Asymmetrically notched specimen: Geometrical characteristics and boundary conditions. The mechanical characteristics are those of steel 1.0553. The truncation order N=15, tolerance parameters $\delta_s = 1{,}99 \times 10^{-5}$ for the series and $10^{-6,2} < \delta_p < 10^{-5}$ for Padé.

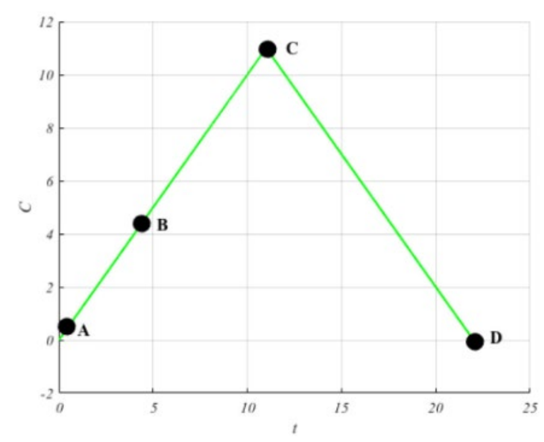

Fig.2: Loading function with for stages of loads A, B, C, and D. In order to assess the efficiency of the algorithm, we specifically investigate its performance in the load-unload case. By subjecting the algorithm to such load variations, we aim to rigorously evaluate its computational effectiveness and robustness in handling load-unload scenarios.

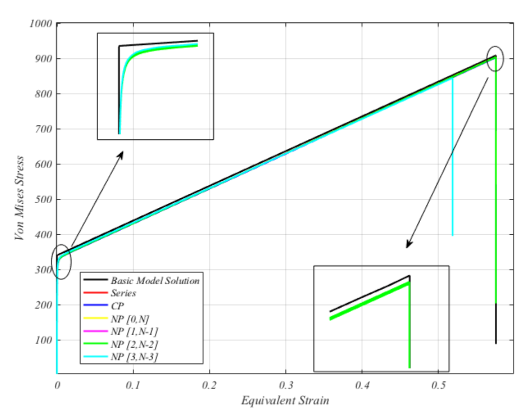

Fig.3: Evolution of von-Mises equivalent stress with respect to the equivalent strain.

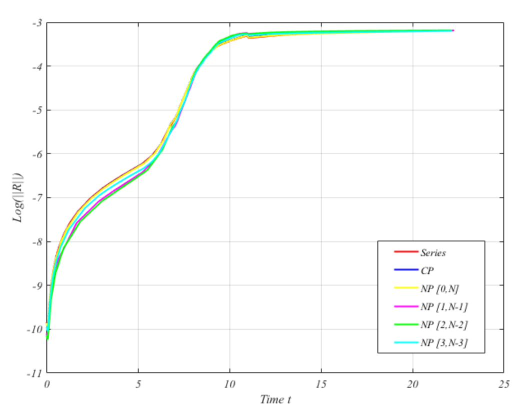

Fig.4: Evolution of residual norm with respect to time for different continuation types

Conclusion and perspectives

This study introduces a regularized elasto-plastic model for analyzing the behavior of an Asymmetrically notched specimen, taking into consideration the coupling of finite transformation and elastoplasticity. The application of Padé approximants in the context of large strain elasto-plasticity is thoroughly discussed in this presentation. Padé approximants, which are rational fractions derived from previously computed Taylor series, are utilized through various techniques. Specifically, this work assesses two types of vectorial Padé approximants, namely the "classical vectorial Padé approximant" and the more recent class of vectorial Padé approximants.

References

1. B. Cochelin, N. Damil, M. Potier-Ferry, Méthode asymptotique numérique, Hermes Lavoissier, 2007.
2. B. Braikat, N. Damil, M. Potier-Ferry, Méthodes asymptotiques numériques pour la plasticité, R. Eur Elém Finis 6 (3) (1997) 337–357.
3. C. El Kihal, O. Askour, Y. Belaasilia, A. Hamdaoui, B. Braikat, N. Damil, M. Potier-Ferry, Asymptotic numerical method for finite plasticity, Finite Elem. Anal. Des. 206 (2022) 103759..
4. El Kihal, C., Askour, O., Hamdaoui, A., Braikat, B., Damil, N., & Potier-Ferry, M. (2022). Convergence acceleration by Padé approximants in the ANM algorithm for large strain elasto-plasticity. Finite Elements in Analysis and Design, 103853.